Quiz 2-Math 201

- Write your name and your I.D. on the booklet
- The duration of the test is one hour
- Calculators are allowed

1. (20 points) a) Find the first four terms of the Taylor series of e^{-x} and $cos(5x)$ centered at $a = 0$.
   b) Compute the limit
   $$\lim_{x \to 0} \frac{e^{-x} - 1 + x}{cos(5x) - 1}$$
   c) Find an approximation of $\int_0^{0.1} \frac{cos(5x)-1}{x} dx$ with an error less than 10^{-4}.(Use the alternating series error estimation theorem)
2. (10 points) Define $f(x) = 1 - x$ if $0 \leq x \leq \pi$ and 0 if $\pi < x \leq 2\pi$. In the Fourier series $f(x) = a_0/2 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$, find a_3.
3. (25 points)Given $f(x, y) = 5xy + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1} y^{2n+1}}{(2n+1)!}$.(Hint: Write the series as a function of x and y)
   a) Find the equation of the tangent plane to $z = f(x, y)$ at the point $P(\pi, 1, 5\pi)$.
   b) Find the directional derivative of f at the point $P_1(\pi, 3)$ in the direction of $v(3, 1)$.
   c) Find the standard linear approximation of $f(x, y)$ at the point $(\pi, 3)$then use to estimate $f(\pi, 2.99)$. Approximate the error.
4. (10 points) Show that $\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+y^2}$ does not exist while $\lim_{(x,y)\to(0,0)} \frac{x^4}{x^2+y^2} = 0$.
5. (20 points)Suppose that over a certain region of space the electric potential V is given by
   $$V(x, y, z) = 5x^2 - 3xy + xyz$$

a) Find the rate of change of the potential at $P(3, 4, 5)$ in the direction of the vector $v = i + j - k$.

b) In what direction does V change most rapidly at P.

c) What is the maximum rate of change at P.

6. (15 points)The length l, width w and height h of a box change with time. At a certain instant the dimensions are $l = 1m$ and $w = h = 2m$, and l and w are are increasing at a rate of $2m/s$ while h is decreasing at a rate of $3m/s$. At that instant, find the rates at which the volume and the surface area are changing. (The box is closed from above.)